

Discrete Markov chains and mixing times

Homework 4

说明

Due 4 月 26 日 (周五) by 17:00

- To submit electronically, email to *both* 舒阳 and Tim (Tim will read your final project proposal)
- To submit a physical assignment, bring to Tim's office (全斋 22) by 17:00 on 4 月 26 日

Please note: Your final project proposal is additional problem 2 and is required to receive credit on this assignment.

1 Textbook problems

- Levin-Peres: 6.6, 7.2, 7.3
- Roch: 5.6, 5.9

2 Additional problems

1. Let $G = (V, E)$ be a non-directed graph without loops and with algebraic connectivity μ_2 . Set $|V| =: n$ and

$$\delta := \min_{v \in V} \deg(v).$$

- (a) We showed that the largest eigenvalue μ_n of the Laplacian matrix L of G satisfies $\mu_n \geq \bar{\delta} + 1$. Use a similar strategy to show that $\mu_2 \leq \underline{\delta} + 3$.
- (b) The bound in (a) is very coarse. By considering the matrix $L - \mu_2(I - n^{-1}\mathbf{1}\mathbf{1}^T)$, show that we in fact have

$$\mu_2 \leq \frac{n}{n-1} \underline{\delta}.$$

- (c) Show that this bound is sharp.

2. *Required for all homework submissions. Your homework will not be accepted if it does not contain this proposal.*

Write a proposal of what you are thinking about doing for your final project for the class. Your proposal must include:

- (a) Whether you will discuss a research project you are working on related to Markov chains and/or graph and/or network theory, or summarize a research paper in these areas.
- (b) If you will write about a paper, which paper(s) are you considering? At this point you may be deciding between two or three, but in the end you will select one.

If you will write about a research project, list some of the relevant research papers. You will need to have a brief discussion of these in your final project.

- (c) Explain your interest in this topic.