# 00100862: 随机过程与统计物理
# Discrete Markov chains and mixing times

Instructor: Tim Mesikepp

Spring 2024
周一：3-4 (10:10-12:00)，单周三：7-8 (15:10-17:00)

## 概述

*Markov chains* are a fundamental tool in modern probability, and their study is rich enough to intersect combinatorics, group theory, representation theory, analysis, and differential geometry. They have seen applications in physics, biology, epidemiology and genetics, to give an incomplete list, and are also ubiquitous in modern computer science.

This course aims to cover the essentials of a modern viewpoint on discrete Markov chain theory, mainly through the lens of the classic Levin-Peres textbook *Markov Chains and Mixing Times*.

## 涵盖的主题

Our provisional course outline is as follows. This material may change due to time constraints and/or instructor and student tastes.

($a$) We will begin by covering the "core chapters" 1-7 of Levin-Peres. This is basically everything a normal human being would want to know about finite Markov chains, and topics include a review of basic concepts and fundamental examples, sampling algorithms, mixing times, couplings, strong stationary times, and lower bound methods.

Our broad goal here will be to understand how to build chains with a given stationary distribution, and then estimate the mixing time of the chain to stationarity as a function of the chain size.

($b$) We will likely proceed to continue in Levin-Peres, for instance to chapters 8, 9,10 and 11, covering random permutations and shuffling, electrical networks, hitting times and cover times. Perhaps also chapter 16 on shuffling problems in genetics, and/or 13,14 and 15 on path couplings, eigenfunctions and the Ising model.

($c$) At any point after ($a$), we may decide to switch to Russell-Peres' beautiful text *Probability on Trees and Networks*. This book's focus is complementary to Peres-Levin, and it covers many interesting topics on infinite trees and networks.

($d$) At any point during $(a)-(c)$ above we may intersperse our considerations with digressions into Roch's *Modern Discrete Probability*.

Students are welcome to submit their preferences for content for parts $(b), (c)$, and $(d)$ to the instructor via WeChat or in-person discussion.

## 教与学方式

The primary goal of lectures will be to communicate big ideas and intuition, although we will also do many detailed proofs. When details are omitted, it is up to the students to carefully read the textbook to see the particulars.

The teacher also appreciates interacting with students during class and will often ask questions, which he hopes will be answered. He furthermore welcomes questions, comments, and objections during lecture time.

# 评分体系

Grades will be *roughly* assigned according to the following scheme:

10%: Class participation
30%: Homework
30%: Presentations
30%: Final paper

## 说明

1. *Class participation.* Do you attend class? Are you engaged? Do you care?

2. *Homework.* Collections of exercises largely drawn from Levin-Peres.

3. *Presentations.* The subsections of Levin-Peres are generally short and amenable to having a student present them in about 30 minutes as part of a normal class. Ideally we will have at least one such student presentation for each chapter we cover, beginning with chapter 2.

   The exact number of presentations per student and the length of each presentation will depend on the number of students. If there are many students, each presentation may just be 15-minute explanation of a certain proof or homework exercise, for example. The sharp lower bound for number of presentations per student is 1. It may be higher.

   Presentations will help students learn the material better than when merely passively receiving it in lecture, and will give a valuable opportunity to practice public speaking.

4. *Final paper.* Towards the end of the semester, each student will survey a research paper in an area of their interest and submit a several-page

summary of the paper, composed in LaTeX, which describes the mains results, their significance, and sketches the arguments.

The bibliographies of the course references below are vast and an excellent starting point for locating interesting research papers.

There will be no midterm of final exams.

## 语言

Lectures will be in English due to the low Mandarin level of the instructor. Students are encouraged to speak with the instructor in Mandarin to give him opportunities to practice. Student presentations may be in either Mandarin or English. The final paper will be English.

## 课本

(*i*) Levin, David and Yuval Peres, *Markov Chains and Mixing Times*, 2nd edition. Available at `https://pages.uoregon.edu/dlevin/MARKOV/`

*Our main textbook.*

(*ii*) Lyons, Russell and Yuval Peres, *Probability on Trees and Networks.* Available at `https://rdlyons.pages.iu.edu/prbtree/`

*Supplementary material, as time allows.*

(*iii*) Roch, Sebastian, *Modern Discrete Probability: An Essential Toolkit.* Available at `https://people.math.wisc.edu/~roch/mdp/`

*Supplementary material, as time allows. Has interesting examples relevant to contemporary data science applications.*